Exercise 36

Find an equation for the plane that passes through $(3, 2, -1)$ and $(1, -1, 2)$ and that is parallel to the line $\mathbf{v} = (1, -1, 0) + t(3, 2, -2)$.

Solution

The equation for a plane is

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0,$$

where $\mathbf{n}$ is a vector normal to the plane and $\mathbf{r}_0$ is the position vector for any point in the plane. The normal vector is perpendicular to both the direction the line goes in, $(3, 2, -2)$, and the displacement vector,

$$(3, 2, -1) - (1, -1, 2) = (2, 3, -3).$$

Take the cross product of these two to get $\mathbf{n}$.

$$\mathbf{n} = (3, 2, -2) \times (2, 3, -3) = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 3 & 2 & -2 \\ 2 & 3 & -3 \end{vmatrix} = (-6 + 6)\hat{\mathbf{x}} - (-9 + 4)\hat{\mathbf{y}} + (9 - 4)\hat{\mathbf{z}} = 5\hat{\mathbf{y}} + 5\hat{\mathbf{z}} = (0, 5, 5)$$

Either of the position vectors, $(3, 2, -1)$ or $(1, -1, 2)$, will do for $\mathbf{r}_0$. Choose $\mathbf{r}_0 = (3, 2, -1)$.

$$(0, 5, 5) \cdot (x - 3, y - 2, z + 1) = 0$$

$$0(x - 3) + 5(y - 2) + 5(z + 1) = 0$$

$$5y - 10 + 5z + 5 = 0$$

$$5y + 5z = 5$$

$$y + z = 1$$